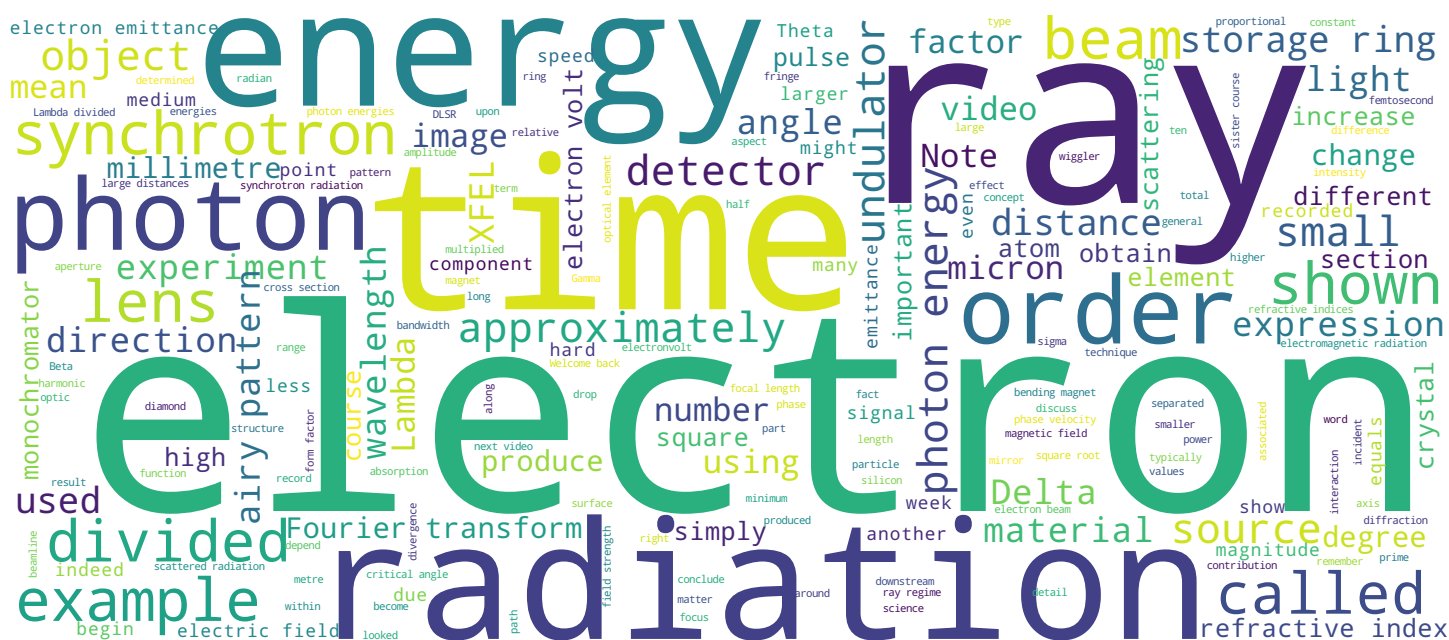


Prof. Philip Willmott



Contents and objectives of this video



- Basic elements of optics theory II
 - Wave (or physical) optics
 - The Airy disc
 - Limits to demagnification

In this video, we will look at aspects of optics that include the fact that light has wavelike properties and is subject to diffraction and interference effects. This means that no barrier or object in the path of electromagnetic radiation will cast a perfect shadow of itself downstream even from a theoretically perfect columnated source. And in fact diffraction effects mean that the further downstream an observer goes from the object the less perfect is the shadow which in fact gradually mutates into the Fourier transform of the transmission function of the object. Now don't panic, we will look at practical examples. which should provide sufficient intuitive insights. What this does mean however is that there are limits to demagnification and how tightly a beam of X-rays, or indeed any other electromagnetic radiation can be focused.

Notes

Summary



0m 05s

Huygens' construction



We begin by considering the approximate geometrical procedure called the Huygens' Construction for determining the propagation of electromagnetic waves. According to this construction, every point of a wavefront in a medium at any instant is the source of secondary spherical wavelets that propagate with the phase velocity of that medium. The sum of all these wavelets conspire to produce the wavefront at a later time. This continues for successive wavefronts in the same manner.

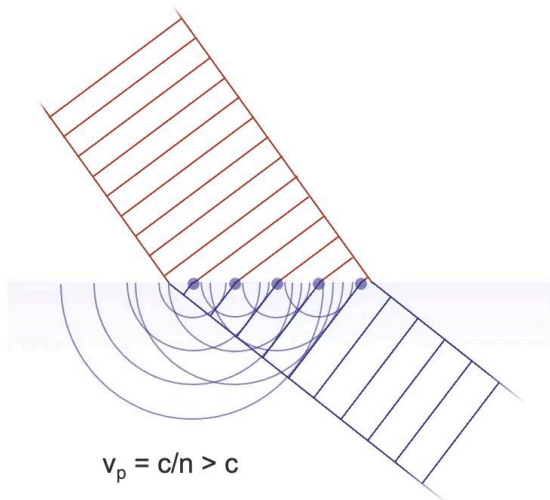
Notes

Summary



1m 14s

Huygens' construction



Huygens' construction can be used to explain refraction. The points where the wavefronts cross the interface between two media act as point sources for radiation travelling with the phase velocity of the medium the wave is entering. In the schematic shown here, the wave goes from vacuum to some medium for which, as in the case of X-rays, the phase velocity is "c" divided by "n", whereby "n" is less than unity, and hence the phase velocity exceeds the speed of light, and the X rays are bent to shallower angles.

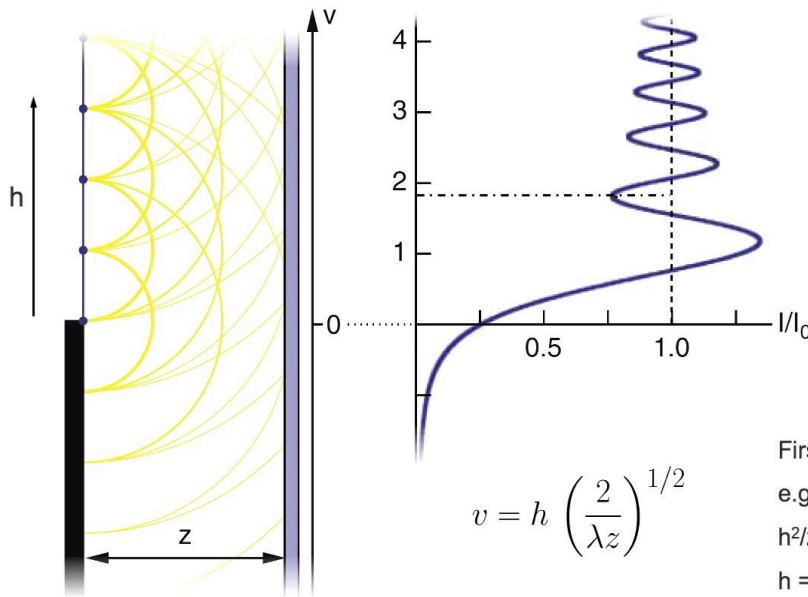
Notes

Summary



1m 57s

Edge diffraction



First minimum @ $v = 1.87$
 e.g. $\lambda = 10 \text{ nm}$
 $h^2/z = 1.752 \times 10^{-8} \text{ m}$
 $h = 100 \text{ } \mu\text{m} \Rightarrow z = 0.57 \text{ m}$

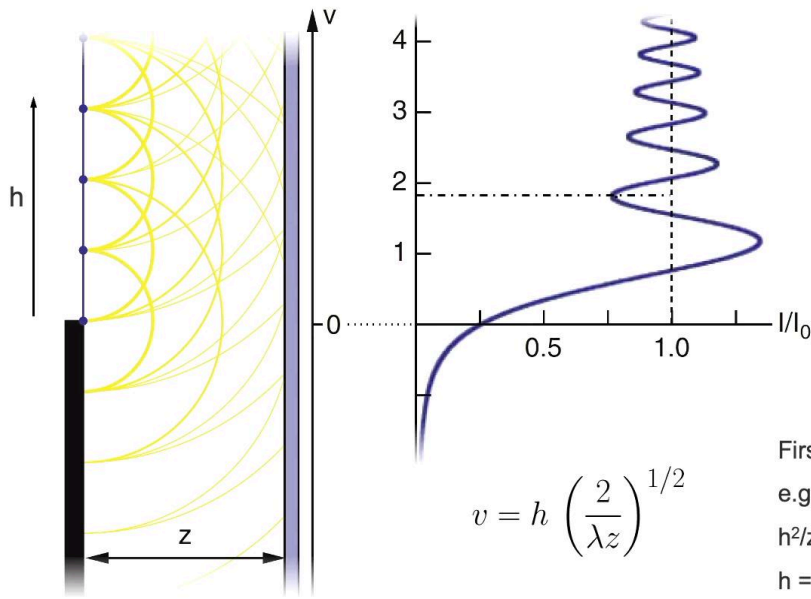
Similarly, we can use Huygen's construction to heuristically explain the pattern on a detector at a distance "Z" from a parallel beam of X-rays partially blocked by an opaque screen. This is not a step function at "z" but instead exhibits interference fringes in the region close to the projection of the edge of the screen. Although it goes beyond the scope of this course to derive an expression for this pattern, we can gain a physical insight using the Huygens' Fresnel Principle. If the propagation of a plane wavefront can be considered to be formed by a row of point emitters as we've just argued, then removing those emitters below the opaque screen means that they cannot contribute to the interference with those wavelets that have not been blocked by the screen. Therefore, the constructed wavefront must deviate from a perfect plane wave, particularly in the neighbourhood of the screen edge. In contrast, the missing contribution to the planer wave for the constructed wave far above the screen is negligible, as here the amplitudes of the blocked wavelets which drop off inversely with the propagation radius are very small. We can use the dimensionless parameter "v" is equal to "h" multiplied by 2 upon Lamda "z" to the half.

Notes

Summary



Edge diffraction



First minimum @ $v = 1.87$

e.g. $\lambda = 10 \text{ nm}$

$h^2/z = 1.752 \times 10^{-8} \text{ m}$

$h = 100 \text{ } \mu\text{m} \Rightarrow z = 0.57 \text{ m}$

It emerges that the first minimum occurs at " v is equal to 1.87. So, for example, 10-nanometre radiation or 124 Electronvolts will exhibit a first minimum at 100 microns if the screen is placed 57 centimetres downstream of the screen. Another example in the hard X-ray regime is also informative. One Angstrom radiation will produce a first fringe with it 42 microns for a screen 10 metres downstream of the sample. Thus, for hard X-rays, edge detection requires detectors with small pixel sizes situated significantly downstream of the sample.

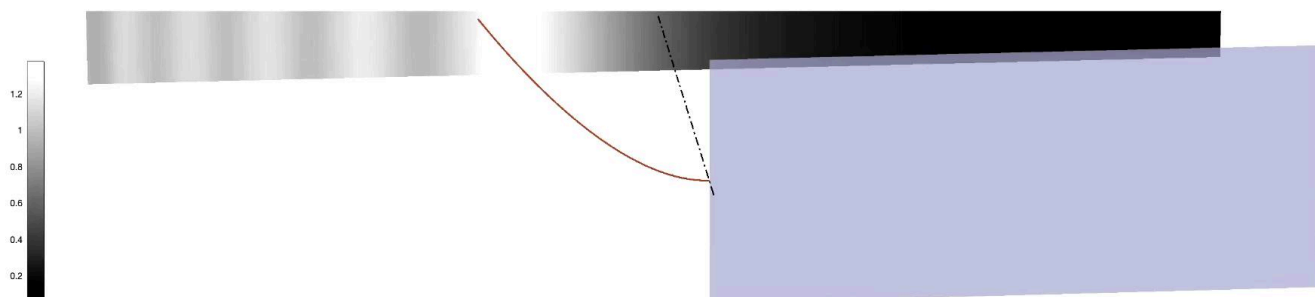
Notes

Summary



4m 26s

Edge diffraction



We can rearrange the expression for " v " to obtain " h " is equal to " v " Λz divided by two to the half. Clearly, the width of the fringe increases with the screen detector distance " z ". We show an animation of this here. Indeed the width increases of the square root of " z " as indicated for the edge of the first fringe by the red curve. This effect is important in synchrotron imaging techniques such as in phase-contrast tomography, which we will discuss in detail in the sister course.

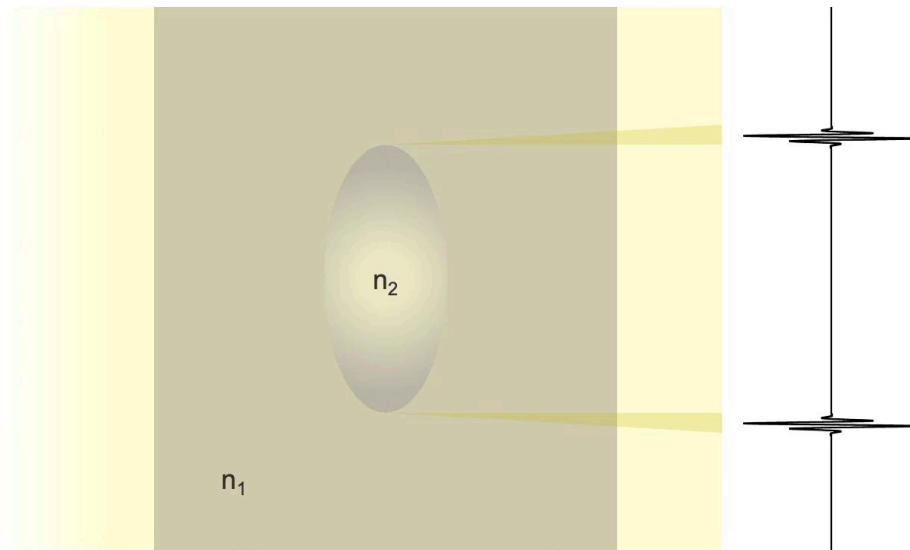
Notes

Summary



5m 16s

Edge diffraction through refraction



It is also important to note that fringes occur not only for opaque screens but also for transparent but heterogeneous objects composed of materials with different refractive indices. Consider an object with the real part of the refractive index " n_1 ", with a component embedded in it with a refractive index " n_2 ", which is marginally larger than " n_1 ". The imaginary components of the refractive indices of both parts of the object are both so small that the object is essentially fully transparent, providing no absorption contrast. The difference in the real part of their refractive indices is, however, sufficiently large to cause an overlap at the border between the two components and a resulting fringe pattern. Now, the upshot of this is that an edge enhanced image is produced if the detector is moved the correct distance away from the sample. Too short a distance and the phase difference between the overlapping radiation is too small to induce observable interference effects. To large a distance and the fringes extend too far laterally potentially smearing out inhomogeneities in the sample that lie close to one another.

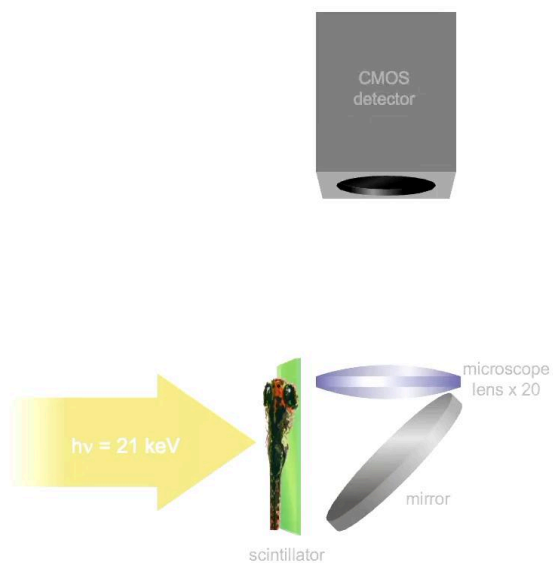
Notes

Summary



5m 55s

Edge diffraction



(c)2018, C.M. Schlepuetz, Swiss Light Source

Courtesy Christian Schlepütz, PSI, and Jörg Huwyler and Emre Cörek, Institute of Pharmaceutical Sciences, Uni Basel.

Now I show this effect in a movie recorded at the [inaudible 00:07:33] beamline of the Swiss Light Source. It shows the change in contrast and clarity of the X-ray transmission image of a zebrafish embryo as a function of distance of the detector from the fishy sample.

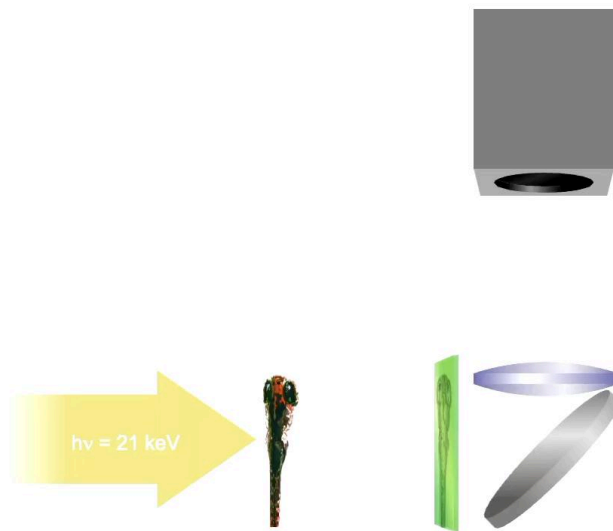
Notes

Summary



7m 28s

Edge diffraction



(c)2018, C.M. Schlepütz, Swiss Light Source
 Courtesy Christian Schlepütz, PSI, and Jörg Huwyler and Emre Cörek, Institute of Pharmaceutical Sciences, Uni Basel.

In the experiment, the scintillator screen, mirror, microscope lens, and sea moss detector are moved gradually downstream of the zebrafish from a distance of 5 millimetres to 50 millimetres. The pair of absorbing features seen in the entire movie are higher density calcium carbonate-based inner ear otoliths. The rest of the anatomy of the embryo is, however, almost perfectly transparent to the 21-kilo electronvolts X-rays used to record this data. Without edge diffraction, the image would have remained featureless. At 30 millimetres, the optimal sample distance, the width of the first fringe can be calculated to be about 1.5 microns. Because the detector has pixels with 6.5 micron linear dimensions, the image from the scintillator plate is magnified using a microscope lens by a factor of 20. This is the basis of face contrast tomography covered in the sister course.

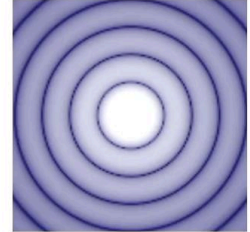
Notes

Summary



7m 53s

Propagation of light through an circular aperture



Airy disc
= FT(circle)

z

$z \gg D^2/\lambda$

Let's see how an image of a circular aperture changes with distance from that aperture. We start with the area detector directly behind the aperture and move it then downstream. We see a similar effect but now the fringes are circular, not very surprisingly given the symmetry. At very large distances for which "z" is significantly larger than the square of the aperture size squared divided by the wavelet of the radiation, what will be recorded is the Fourier transform of the circular transmission function. This is called the airy function. So for example if the aperture 10 microns in diameter, and two Angstrong radiation is used, the airy function will be recorded for distances significantly larger than half a metre.

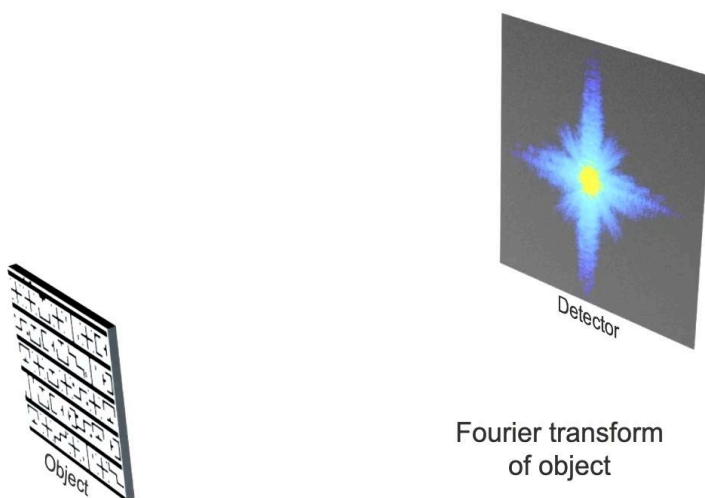
Notes

Summary



9m 08s

Before we continue. What is a lens?



Courtesy I. Mochi, PSI

So as I've mentioned, now, a few times the patterns produced by scattering radiation by an object if it is recorded at sufficiently large distances is simply the Fourier transform of that object. So with this in mind, let us ask the question, what is a lens?

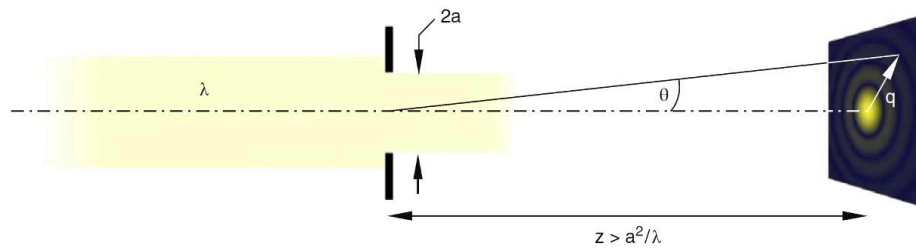
Notes

Summary

10m 09s



Limits to focusing light



If we now capture the scattered radiation with a focusing lens, a detector placed in the focal plane of that lens will record an image of the original object. This leads us to conclude that a lens performs an inverse Fourier transform on the scattered radiation it collects. What a very clever device a lens is. Consider again, a perfectly monochromatic, parallel and uniform plane wave of light passing through a circular aperture of diameter "d" is equal to "2a". The transmitted pattern will appear to be a well-defined disc with the same dimensions on a detector, but only if this detector is placed immediately behind the aperture. But as we have said, as a detector is moved further downstream, the edges of the discs become modulated until at large distances a pattern is recorded. That is, in fact the square of the Fourier transform of the aperture called the Airy Pattern. The radial profile of an airy pattern has its first minimum when " $ka \sin \theta$ " is equal to 3.8317. But we should remember that " k " is equal to $2\pi / \lambda$ and " d " is equal to "2a". We see that the minimum subtends an angle θ given by $\sin \theta$ is equal to $1.22 \lambda / d$.

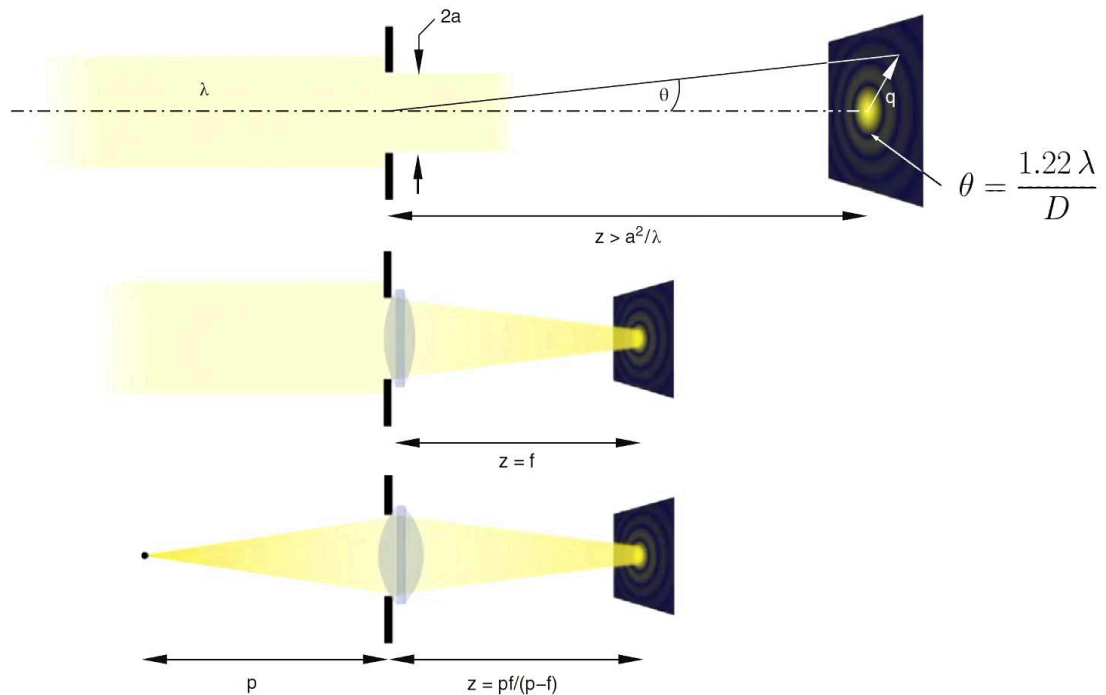
Notes

Summary

10m 31s



Limits to focusing light



Because Theta is generally very small, we can simplify this equation and state that Theta is equal to 1.2 to Lambda divided by "d", Theta being expressed in Radians. The area encompassed by this first fringe minimum is called the airy disc. Now let's consider a situation as depicted in the middle image in which a perfect operation-free converging lens is placed directly behind the aperture. This lens, as we know, produces the Fourier transform of the scattered radiation. But because the lens is directly buttered up to the aperture, we therefore conclude that it will produce the airy pattern as in the upper figure, but now not at the large distances 'z', but at the focal plane of the lens "f". Finally, in the third image, let's consider a lens imaging a point source. Once again, the image will be the airy pattern, which by definition is necessarily larger than the source.

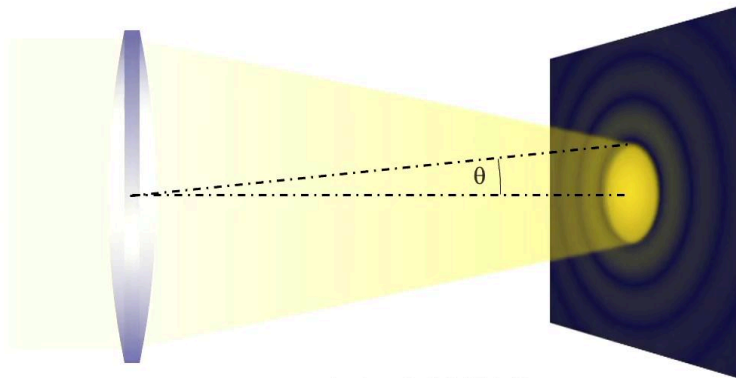
Notes

Summary

12m 10s



Limits to focusing light



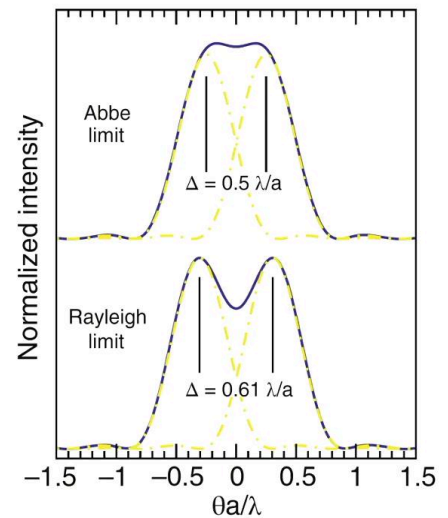
$$\sin \theta = 3.8317 \lambda / 2\pi a$$

$$\Rightarrow \sin \theta = 1.22 \lambda / D$$

$$\text{e.g. } D = 1 \text{ mm}, \lambda = 5 \text{ \AA}$$

$$\Rightarrow \sin \theta = \theta = 6.1 \times 10^{-7}$$

$$\text{e.g. } z = f = 100 \text{ mm, minimum spot} = \phi 12.2 \text{ nm}$$



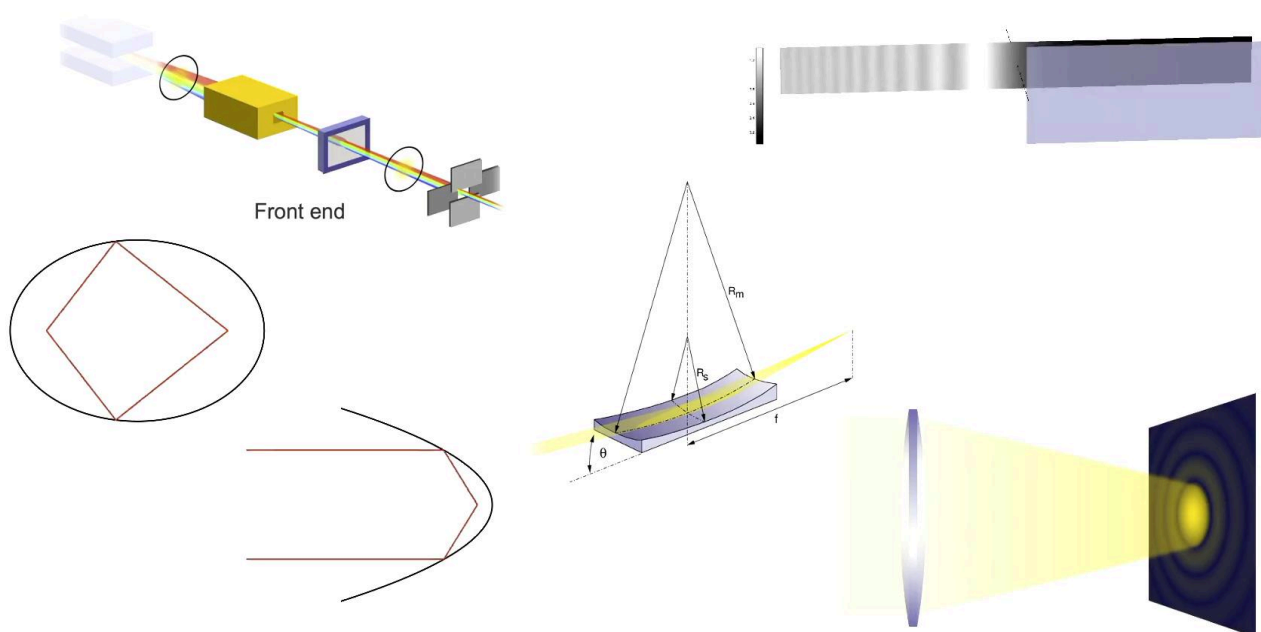
The ultimate spatial resolution of a lens is thus determined by the airy pattern. If we have two features separated by a certain distance from one another, their image using a lens will only resolve them if their airy patterns do not overlap too closely. Abbe defined this minimum through the dimensions parameter Theta "a" divided by Lambda as being equal to the angular separation between two points. To be Delta is equal to Lambda divided by "2a" resulting in the two airy patterns being separated by their full width half maximum. Rayleigh defined it slightly differently as being at Delta equals 0.61 Lambda divided by "a" that is, the airy pattern being separated by the airy disc radius. So we conclude that we cannot arbitrarily demagnify a source using a lens as the lens itself induces diffraction effects that will smear the source. For example, a one-millimetre diameter lens with a focal length of 100 millimetres can focus five Angstrong radiation down to a minimum spot size of just over 12 nanometres.

Notes

Summary



Summary of this section



To summarise this section, we first looked at the most important components of beamline front ends before considering focusing using elliptical and parabolic optical elements and assuming geometrical array optics. We looked at the equations used to determine the bending radius of cylindrical or spherical approximations to elliptical and parabolic optics. We then took into consideration the wavelet nature of electromagnetic radiation and how diffraction results in fringes when an object is placed in the path of a beam. This led us to conclude that there is a lower limit to demagnification given by the numerical aperture or collecting angle of an optical element given by the so-called airy patterning.

Notes

Summary



14m 43s

In the next section...



We have now covered the necessary theoretical background material for X-ray optics. This is important, so I recommend you go over this and the previous video carefully and perhaps also take a look at the original material in my textbook. In the next section, we begin to look at the function of primary optics, namely mirrors and monochromators.

Notes

Summary



15m 46s